Formation of Periodic Ripples in a Surface Skin

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A solid elastic skin on a liquid surface aquires a periodic ripple formation when a compressive strain surpasses a critical value. From a calculation the ripple wavelength is found to be proportional to the 3/4th power of the skin thickness. This instability can be described as a kind of second order phase transition, where a relative amplitude of the ripple wave is the order parameter. In addition, when the skin area is abruptly compressed the ripple wavelength depends on the magnitude of the compressive strain. Examples for skin rippling with wavelengths between 10 μm and 100 m are discussed.

§ 1. Introduction

When we apply an oil paint layer too thickly upon a horizontal plane we may obtain a rippled surface (Figure 1). In some kinds of paint this effect is even desirable and can also be produced in thin coatings. The rippling is shaped very regularly with a definite wavelength. As is well known the oil layer hardens from above due to a process by which oxygen absorbed from the air causes a polymerization of the linseed-oil and an increase in volume. This volume increase can only take place normal to the layer. Dilation within the plane of the layer however is prevented and therefore a high compressive stress is set up which gives the thoroughly polymerized layer a special hardness. In thicker layers the uppermost hardened skin becomes unstable under the compressive stress and buckles into regular corrugations.

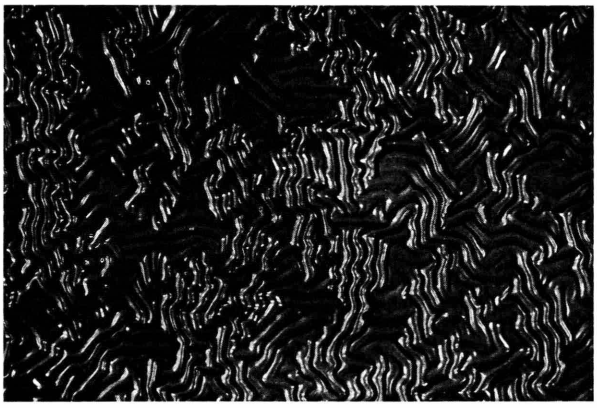

Fig. 1. Rippled surface of oil plaint layer, too thickly applied upon horizontal plane, after drying. Average ripple wavelength 1.0 mm.

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How can we understand this periodical ripple formation? Layers of oil paint are not ideal objects for this investigation because of the time dependent skin thickness. Simpler conditions are met with a thin plastic film upon the surface of water. By a soft uniaxial compressive strain a unidirectional periodic corrugation can be easily produced (Figure 2).

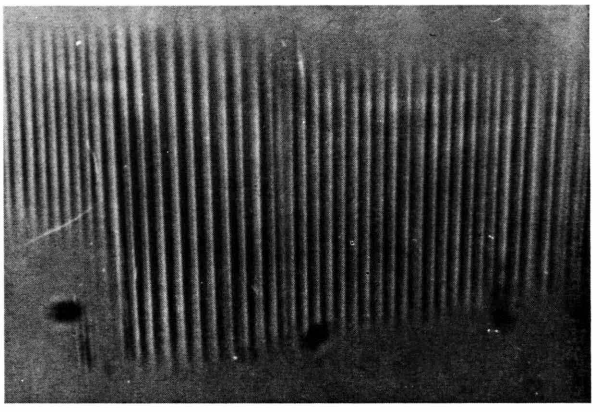

Fig. 2. Periodic corrugation produced by unidirectional compressive strain of a collodian film layed on the surface of water. Areas with different wavelengths caused by inhomogeneous film thickness. Larger wavelength 18 μm, smaller wavelength 13 μm.

To the aforementioned examples others can be added. For instance also the recently described ripple formation by compact welding [1] might be based on a similar process. A thin oxide layer could be responsible for the rippling. Between the two metal parts the layer experiences a compressive strain due to the impact. While the metal will flow plastically the oxide layer ripples because of its higher yield-point. Another example is the corrugation of the asphalt bed observed on some streets near the stop sign. Obviously the upper, more solid, part of the bitumen layer undergoes a compressive straing by the braking action of vehicles hereby corrugating with a uniform wavelength. The essential factor governing the length of the waves should be the thickness of the solid elastic skin. Thus large waveforms are to be expected from the compressive strain of the solid earth crust and short wavelength ripples should appear in solid molecular films.

It is the purpose of this paper to explore the conditions of this ripple formation and to present formulae which contain the determining parameters. Preliminary results which prove the relation between thickness and critical wavelength will be presented at the end of this paper. More extended experimental results shall be reported in a later paper.

§ 2. A Model for Static Skin Ripples

We consider a plane elastic, solid skin of thickness d layed on the surface of a liquid (x-y plane). A uniaxial compressive stress in x direction may reduce the length L by ΔL. Hereby no other stresses shall appear in y and z direction so that the skin will slightly increase in cross section. We neglect any flow in the adjoining liquid. As long as the skin stays planar we only get elastic strain energy. If however the skin develops a sinusoidal rippling with the wavelength $\lambda = 2\pi/k$, where k is the wavenumber, then other energy terms have to be added. Besides an elastic bending energy there is a gravity term and a term for surface tension. The stored energy per unit-area f averaged over one wavelength is found to be

$$f(k, \zeta_0^2) = \frac{E\,d}{2}\left[\left(\frac{1}{4}k^2\zeta_0^2 - \frac{\Delta L}{L}\right)^2 + \frac{d^2 k^4 \zeta_0^2}{24}\right] + \frac{1}{4}(\varrho\, g + \gamma\, k^2)\zeta_0^2\,. \quad (1)$$

In this equation ζ_0 means the amplitude of the ripple. We always presume ζ_0 to be small compared to λ. Also $k\,d < 1$ may be fulfilled. In the two elastic terms E means Young's modulus of the skin. In the gravity term ϱ means the density of the liquid and g the acceleration due to gravity. With γ we mean the sum of surface tensions of both the upper and lower side of the skin. When $\zeta_0 = 0$ we obtain the case of the plane skin with pure strain energy.

The equilibrium amplitude z_0 will be reached when f becomes a minimum. For getting this amplitude we have to form

$$(\partial f/\partial \zeta_0^2)_{\zeta_0 = z_0} = 0 \quad (2)$$

which gives us

$$z_0^2 = \frac{4}{k^2}\left(\frac{\Delta L}{L}\right) - \frac{d^2}{3} - \frac{4\,\gamma}{k^2\,E\,d} - \frac{4\,\varrho\,g}{k^4\,E\,d} \geqq 0\,. \quad (3)$$

For those pairs of values $(\Delta L/L, k)$ which give $z_0^2 < 0$ we have the plane skin with $z_0^2 \equiv 0$ as the stable solution. Equation (3) with $z_0^2 = 0$ will give us the minimum of $\Delta L/L$ needed for rippling the skin as a function of k.

$$\left(\frac{\Delta L}{L}\right)_0 = \frac{k^2\,d^2}{12} + \frac{\gamma}{E\,d} + \frac{\varrho\,g}{E\,d\,k^2}\,. \quad (4)$$

By increasing $\Delta L/L$ slowly from zero we obtain a minimum of $(\Delta L/L)_0$ at which the ripple formation starts with a well defined wavenumber k_c. You get this critical wavenumber by differentiating and zeroing $(\Delta L/L)_0$ as a function of k^2 in Eq. (4):

$$k_c = (12\,\varrho\,g/d^3\,E)^{1/4}\,. \quad (5)$$

For k_c the minimum of $(\Delta L/L)_0$ is

$$(\Delta L/L)_c = (\varrho\,g\,d/3\,E)^{1/2} + (\gamma/E\,d)\,. \quad (6)$$

Once more $(\Delta L/L)_c$ as a function of the skin thickness d gives the minimum value

$$(\Delta L/L)_c^* = (9\,\gamma\,\varrho\,g/4\,E^2)^{1/3} \quad (7)$$

at the skin thickness

$$d^* = (12\,\gamma^2/\varrho\,g\,E)^{1/3}\,. \quad (8)$$

For this thickness the critical wavenumber k_c from Eq. (5) is

$$k_c^* = (\varrho\,g/\gamma)^{1/2}\,. \quad (9)$$

By taking z_0^2 out of Eq. (3) into Eq. (1) the stored energy per unit-area $f(k)$ for the static case can be presented in the following way

$$f(k) = \frac{E\,d}{2}\left[\left(\frac{\Delta L}{L}\right)^2 - \left(\left(\frac{\Delta L}{L}\right) - \left(\frac{\Delta L}{L}\right)_0\right)^2\right]. \quad (10)$$

Here $(\Delta L/L)_0$ has been used from Equation (4). This only holds for the rippled skin. In the other case of the plane skin with $z_0 \equiv 0$ we obtain from Eq. (1)

$$f_0 = \frac{E\,d}{2}\left(\frac{\Delta L}{L}\right)^2. \quad (11)$$

Now we can also answer the question of which wavenumber of rippling for a given $(\Delta L/L)$ will be formed statically. We have to look for the minimum of f_1 as a function of k^2. This means

$$\frac{\partial f}{\partial k^2} = \frac{\partial f}{\partial (\Delta L/L)_0} \cdot \frac{\partial (\Delta L/L)_0}{\partial k^2} = 0 \,. \quad (12)$$

When $\partial f/\partial (\Delta L/L)_0 = 0$, then we find $(\Delta L/L)_0 = (\Delta L/L)$. But here $f(k)$ has a maximum relative to k^2.

When on the other hand $\partial (\Delta L/L)_0/\partial k^2 = 0$ then Eq. (5) again holds, this time namely for any fixed $(\Delta L/L) > (\Delta L/L)_c$. Here now $f(k)$ has a minimum relative to k^2. We define $f_{\min}(k) = f_1$ and obtain

$$f_1 = \frac{E\,d}{2}\left[\left(\frac{\Delta L}{L}\right)^2 - \left(\left(\frac{\Delta L}{L}\right) - \left(\frac{\Delta L}{L}\right)_c\right)^2\right]. \quad (13)$$

The corresponding amplitude is obtained from Eqs. (3), (4) and (6):

$$z_c = \frac{2}{k_c}\left[\left(\frac{\Delta L}{L}\right) - \left(\frac{\Delta L}{L}\right)_c\right]^{1/2}. \quad (14)$$

In Figure 3 the stored energy per unit-area f is plotted as a function of $(\Delta L/L)$. Below $(\Delta L/L)_c$

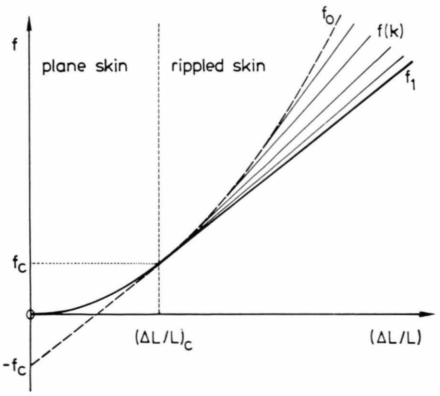

Fig. 3. Stored energy per unit-area f as a function of compressive strain $(\Delta L/L)$. f_0 from Eq. (11), f_1 from Eq. (13) and $f(k)$ from Equation (10).

the plane skin is stable. According to Eq. (11) f_0 increases quadratically in $(\Delta L/L)$. According to Eq. (10) the k dependent solutions of the rippled skin give a linear dependence between $f(k)$ and $(\Delta L/L)$. These straight lines are tangents to the parabola of Equation (11). f_1 from Eq. (13) has its point of contact at $(\Delta L/L)_c$. For $(\Delta L/L) < (\Delta L/L)_c$ the amplitude of rippling comes out to be purely imaginary. At $(\Delta L/L)_c$ a kind of second order phase transition takes place. The ripple amplitude continuously increases from zero when $(\Delta L/L)$ becomes larger than $(\Delta L/L)_c$.

For the ripple phase an order parameter p may be defined. Let us choose p as the ratio $z_c/z_{c,m}$ of two amplitudes of the rippled skin. z_c is the equilibrium amplitude from Equation (14). $z_{c,m}$ would be the amplitude of an ideal rippled skin being infinitely thin ($d = 0$) without a liquid below ($\varrho = 0$) and without surface tension ($\gamma = 0$) but with the same ripple wavelength. This would be the maximum amplitude of a completely stress free skin. We get

$$z_{c,m} = \frac{2}{k_c}\left(\frac{\Delta L}{L}\right)^{1/2}. \quad (15)$$

So we have

$$p = \frac{z_c}{z_{c,m}} = \sqrt{1 - \frac{(\Delta L/L)_c}{(\Delta L/L)}} = \left(1 - \frac{1}{q}\right)^{1/2} \quad (16)$$

when

$$q = (\Delta L/L)/(\Delta L/L)_c \quad (17)$$

represents a normalized strain parameter.

Figure 4 presents the graph of the order parameter p as a function of q. For $q = 1$ the order parameter starts increasing from zero with an infinite slope. p has reached about 70% at $q = 2$ and about 90% at $q = 5$. The order parameter could also be used for $k \neq k_c$ when we replace $(\Delta L/L)_c$ by $(\Delta L/L)_0$.

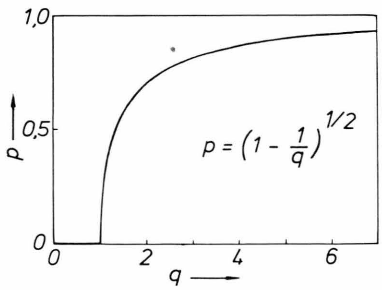

Fig. 4. Order parameters p versus reduced strain parameter q.

§ 3. Ripple Formation by Sudden Compressive Strain

As long as we increase the strain slowly enough that at each moment the skin is in a state of equilibrium we find that the ripple wavenumber k_c is independent of the magnitude of $(\Delta L/L)$. We shall see that by suddenly applying a constant overcritical $(\Delta L/L)$ the ripple wavenumber k_f which first appears will depend on the magnitude of $(\Delta L/L)$. This leads to a hydrodynamical problem of the flow of the liquid coupled to the ripple formation. Let us consider very small Reynolds numbers so that we can neglect the inertial term.

For the present let us give a solution for infinite depth of the liquid. Let $p(x,z)$ be the pressure in the liquid (without the hydrostatic term) and may $u(x,z)$ and $w(x,z)$ be the x and z component of the velocity field $\boldsymbol{v}$ of the liquid, where the initially plane liquid surface may be at $z=0$. Then the solution is

$$\begin{aligned} p(x,z) &= p_0\, e^{kz} \sin k z\,, \\ u(x,z) &= u_0\, k\, z\, e^{kz} \cos k z\,, \\ w(x,z) &= w_0 (1-k z)\, e^{kz} \sin k z\,. \end{aligned} \tag{18}$$

This solution fulfills the boundary condition $u(x,0)=0$ which is required because of the adhesion of the liquid to the overlying skin.

Because of $\operatorname{div} \boldsymbol{v}=0$ (incompressible fluid) we have

$$u_0 = -w_0\,. \tag{19}$$

Furthermore from the force equations, for instance

$$\frac{\partial p}{\partial x} = \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) \tag{20}$$

where μ is the viscosity of the liquid, we get a relation between the pressure amplitude and the velocity amplitude

$$p_0 = 2\,\mu\, k \cdot u_0\,. \tag{21}$$

The next will be to formulate a relation between the pressure amplitude p_0 and the wave amplitude ζ_0 of the skin. As soon as the skin begins to form the sine wave it will develope and maintain forces to the adjacent liquid until the ripple amplitude has reached the equilibrium value $z_0(k)$ in Equation (3).

Hereby the differential work performed by the skin to the liquid is

$$\delta A = p(x,0)\, \mathrm{d}V = p_0 \sin k x \cdot y\, \mathrm{d}x \cdot \mathrm{d}\zeta\,. \tag{22}$$

We use $\zeta = \zeta_0 \sin k x$ and average over x. Then this averaged work per unit-area has to be equal to the decrease of the stored energy per unit-area of the skin.

$$\mathrm{d}f = \tfrac{1}{2}\, p_0\, \mathrm{d}\zeta_0\,. \tag{23}$$

In addition the z component of the velocity field next to the skin has to be equal to the time derivation of ζ

$$\mathrm{d}\zeta_0/\mathrm{d}t = w_0\,. \tag{24}$$

Finally from Eq. (23) and (24) we obtain the following differential equation

$$\left(1 - \frac{\zeta_0^2}{z_0^2}\right)^{-1} \cdot \frac{\mathrm{d}\zeta_0}{\zeta_0} = \frac{\mathrm{d}t}{\tau} \tag{25}$$

where we have used the reduction

$$\tau = 8\,\mu/\sigma\, \mathrm{d}k^3\, z_0^2\,. \tag{26}$$

The general solution of Eq. (25) is

$$\frac{\zeta_0}{(z_0^2-\zeta_0^2)^{1/2}} = \frac{\zeta_{00}}{(z_0^2-\zeta_{00}^2)^{1/2}} \cdot \exp\{t/\tau\}\,. \tag{27}$$

Hereby ζ_{00} means the amplitude at $t=0$.

We see that for $\zeta_{00} \ll z_0$ the amplitude ζ_0 increases exponentially with time. The equilibrium amplitude z_0 is approached with the relaxation time $\tau' = 2\,\tau$. According to Eq. (26) and (3) τ is k dependent.

The initial amplitude ζ_{00} is determined by thermal fluctuations of the skin in the state $(\Delta L/L)=0$. Let us look at Eq. (1) again. That part of the energy of the whole skin (with its area $y \cdot L$) which is quadratic in ζ_0 must be equal to $\tfrac{1}{2} k_B T$ in the thermal average. So we get

$$\zeta_{00}^2 = \langle \zeta_0^2 \rangle = \frac{2\, k_B T / y\, L}{\frac{1}{12}\,\sigma\, d^3 k^4 + \gamma\, k^2 + \varrho\, g}\,. \tag{28}$$

We notice the contributions of gravity, surface tension, and bending as well as their different k dependence. Directly after the compressive strain $(\Delta L/L)$ is being applied, every k mode will be amplified with its own time constant $\tau(k)$. With a high probability the k mode with minimum τ will succeed and withdraw from other modes the energy they already had gained.

Thus the thermally conditioned initial amplitude plays a secondary roll. By differentiating τ after k and zeroing we find for the most probable k_f:

$$\tfrac{1}{2}\, d^2 k_f^2 = \left(\frac{\Delta L}{L} - \frac{\gamma}{E\,d}\right) + \left[\left(\frac{\Delta L}{L} - \frac{\gamma}{E\,d}\right)^2 + \frac{\varrho\, g\, d}{E}\right]^{1/2}. \tag{29}$$

For $(\Delta L/L) = (\Delta L/L)_c$ we have $k_f = k_c$. And for small deviations from this limiting value one gets the linear approximation

$$\left(\frac{\Delta L}{L}\right) - \left(\frac{\Delta L}{L}\right)_c = \frac{1}{3}\, d^2 (k_f^2 - k_c^2)\,. \tag{30}$$

So for an overcritical $(\Delta L/L)$ we expect k_f always larger than k_c. The questions arises if afterwards k_f will transform into k_c. This will depend on the boundary conditions for the skin in the y-x range and shall not be discussed here further. Also the question of the relaxation time for optimizing the wavenumber, in case the boundary conditions should allow, may go unanswered.

The functional relations between $(\Delta L/L)$ and k^2 given in Eqs. (4), (5), and (29) can be brought into a consistent reduced form. Let us use

$$r = k/k_c \tag{31}$$

$$\tilde{d} = d/d^* \tag{32}$$

$$\alpha = 2 + \tilde{d}^{-3/2} . \tag{33}$$

With q from Eq. (16), Eq. (4) gets the reduced form

$$\alpha(q_0 - 1) = (r - 1/r)^2 \tag{34}$$

and from Eq. (29) we obtain

$$\alpha(q_f - 1) = (r - 1/r)^2 + 2(r^2 - 1/r^2) . \tag{35}$$

In Fig. 5 the graphs of these functions are presented. The graph o separates the "plane phase"

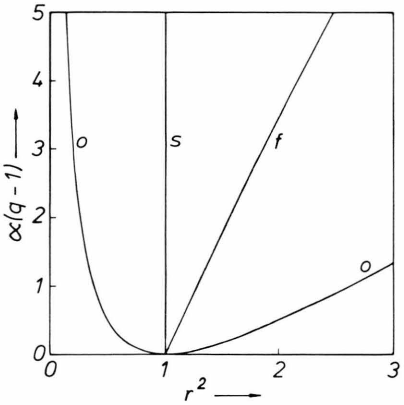

Fig. 5. Representation of relations between strain and wavenumber in a reduced form. For details see text.

from the "rippled phase" [Equation (34)]. The graph s (slow) holds when increasing q slowly [Equation (5)]. The graph f (fast) gives the value of r^2 when a constant q is being applied instantaneously [Equation (35)].

Also Eq. (6) can be converted into the reduced form

$$3\,\tilde{q}^{-1} = 2\,\tilde{d}^{1/2} + \tilde{d}^{-1} \tag{36}$$

where

$$\tilde{q} = q/q^* = (\Delta L/L)_c^* / (\Delta L/L)_c = (\widetilde{\Delta L/L})_c^{-1} . \tag{37}$$

The case of a finite depth h of the liquid can be treated in the same way. However the mathematical solution of the flow equations with the required boundary conditions $u(x, 0) = 0$, $u(x, -h) = 0$ and $w(x, -h) = 0$ is no more so simple as in Equation (18). Its representation can be dispensed with because further calculations give a very complicated k dependence of the relaxation time τ. It can be seen from Eq. (18) that pressure p and velocity v decrease exponentially with distance from the liquid surface. Therefore a depth $h \geqq \lambda_c$ practically shows no difference to an infinite depth of liquid.

§ 4. Discussion of Examples

The most important relation is given by Equation (5). The ripple wavelength is directly proportional to the 3/4th power of the skin thickness. Only the density ϱ of the liquid and Young's modulus E of the skin are needed as parameters. E varies between about 10^{11} N/m² (typical for metals) and about 10^7 N/m² (typical for rubber)[3]. For these limiting cases you will find in Fig. 6 two graphs of λ_c vs. d

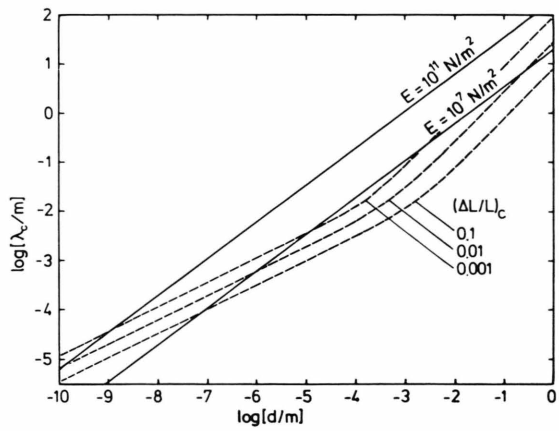

Fig. 6. Ripple wavelength λ_c versus skin thickness d in a double logarithmic plot using $\varrho = 10^3$ kg/m³ (H_2O). Full curves: Young's modulus E as parameter. Dashed curves: Critical strain $(\Delta L/L)_c$ as parameter.

in a double logarithmic plot. For ϱ the density of water has been chosen. A 20 times higher density would only redouble λ_c. Also the various E values can change λ_c by not more than a factor of 10. The surface tension γ has no influence at all.

From Fig. 6 we find out that a molecular skin thickness of $d = 1$ nm has a ripple wavelength of about 10 μm, that means $\lambda_c : d = 10^4$. In the other extreme case of a skin thickness of $d = 1$ km one still gets $\lambda_c : d = 10$. Here we are in the range of fold-mountains of the solid crust of the earth formed by the continental shift. At this limit rheological problems become also involved [2] as we shall see below.

The ripple wavelength of the asphalt bed near the stop sign of a motor-street is about 30 cm. It should be caused by a skin of some millimeters of thickness, which has been formed by rolling fine slag into the surface and by drying up. The rippled surface of the hardened oil paint in Fig. 1 has a wavelength of about 1.0 mm. From Fig. 6 we find that the rippling was formed when the polymerized film had a thickness of less than 1 μm.

A determination of E out of λ_c and d requires high precision because the relative error in E will be four times the relative error in λ_c plus three times the relative error in d. With only an approximate knowledge of Young's modulus E, however, one can determine λ_c from d or d from λ_c quite well. From Fig. 2 we can find wavelengths λ_c of 13 and 18 μm. With a Young's modulus $E = 1.4 \times 10^9$ N/m^2 for bulk collodian [3] we calculate skin thicknesses d of 1.2 and 1.8 nm. This however seems to be too small. We intend to measure d independently and then calculate E. It remains to remark that for a uniform strain of the skin in two dimensions, as in case of Fig. 1, the E-modulus has to be replaced by $E/(1-\nu)$, where ν is the Poisson number.

Now let us pay more attention to $(\Delta L/L)_c$ from Equation (6). It gets a minimum value [Eq. (7)] for a skin thickness d^* [Equation (8)]. Here also the surface tension is involved. When we use as representative values $E = 10^9$ N/m^2, $\varrho g = 10^4$ N/m^3, and $\gamma = 10^{-1}$ N/m then $d^* = 2.3 \times 10^{-5}$ m and $(\Delta L/L)_c{}^* = 1.3 \times 10^{-5}$. By using Eq. (36) we obtain for the very small skin thickness of $d = 10^{-9}$ m a $(\Delta L/L)_c \approx 10\%$ and for the very large skin thickness of $d = 10^3$ m a $(\Delta L/L)_c \approx 6\%$ (see also Fig. 6, dashed curves). For many materials, however, these large critical values mean that one reaches the yield-point beforehand and rheological questions become important [2]. So the range of validity of the theory has to be reduced from both sides. Otherwise those complications arising from passing over the elastic limit require special considerations.

In order to keep the skin in a state in which it is compressed by ΔL of its length L we have to apply a force

$$K = L\, y\, \partial f / \partial \Delta L \qquad (38)$$

along the width y. Let us call

$$\Gamma = K/y = \partial f / \partial (\Delta L/L) \qquad (39)$$

the skin tension. Then for the plane skin we derive from Eq. (10)

$$\Gamma_0 = \partial f_0 / \partial (\Delta L/L) = E\, d \cdot (\Delta L/L) \qquad (40)$$

and for the rippled skin we derive from Eq. (13) and from Eq. (6)

$$\begin{aligned} \Gamma_1 = \partial f_1 / \partial (\Delta L/L) &= E\, d\, (\Delta L/L)_c \qquad (41) \\ &= (\varrho\, g\, E\, d^3/3)^{1/2} + \gamma\,. \end{aligned}$$

We see that first the skin tension Γ increases directly proportional to $(\Delta L/L)$ and then it stays constant when $(\Delta L/L)$ becomes larger than $(\Delta L/L)_c$.

In the practical experiment performed on a skin layed freely upon a liquid surface, simultaneously with the compression of the film area by $\Delta L \cdot y$ we have to generate new free liquid surface of the same size. When γ_0 means the surface tension of the liquid then already without external forces one observes an intrinsic skin tension

$$\Gamma_i = \gamma - \gamma_0 \qquad (42)$$

and this happens in both dimensions of the skin plane. Therefore again we have to replace E by $E/(1-\nu)$ in Equation (40). So the solid film on the liquid surface gets an initial compressive strain of

$$(\Delta L/L)_i = (\gamma - \gamma_0)\,(1-\nu)/E\, d \qquad (43)$$

which when becoming larger than $(1-\nu) \cdot (\Delta L/L)_c$ would yield the complete collapse of the solid skin. But this cannot happen because the external critical tension $\Gamma_{krit} = \Gamma_1 - \Gamma_i = (\varrho\, g\, E\, d^3/3)^{1/2} + \gamma_0$ would need to be negative.

In all our considerations we have made the restrictive assumption that the amplitude ζ_0 or z_0 has to be small compared to the wavelength. Otherwise the statement that the rippling was sinusoidal would no longer be valid. With increasing amplitude the rippled skin can degenerate into a meandering shape. Then it comes to a special instability which can easily be observed in strained films of collodian on water (Figure 7). One trough of the ripple wave will grow in depth at the expenses of the rest of the wave so that the rippling will disappear completely. What remains is a defect with the shape of a straight line. Here the skin has been folded up in order to form a double layer.

A special discussion is required for understanding the ripple formation by compact welding [1]. If we

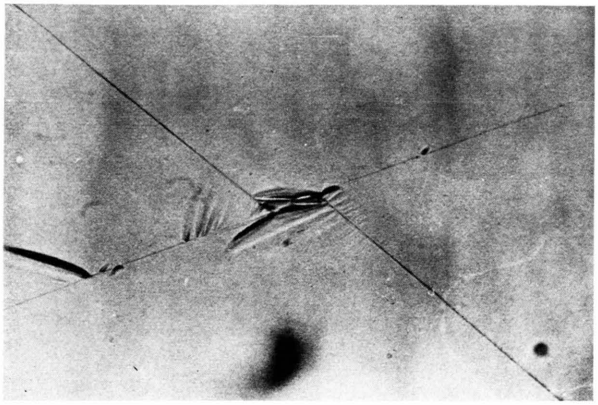

Fig. 7. Skin folding by transient overcritical amplitudes of the ripple. Double layers hanging in the liquid along straight lines.

suppose that the rippling comes from an oxide (or some other) layer between the two metal parts which shall be welded together then the gravity term has to be dropped. But this way the ripple wavelength from Eq. (5) would become infinite. Nevertheless we are able to understand the finite ripple wavelength considering the compact welding as a very fast event. Therefore we have to apply Equation (29). By dropping the gravity term we get

$$k_{\mathrm{f}} = \frac{2}{d} \cdot \left(\frac{\Delta L}{L} - \frac{\gamma}{E\,d} \right)^{1/2}. \qquad (44)$$

The ripple wavelength will be the smaller the more $(\Delta L/L)$ prevails the critical strain $(\Delta L/L)_{\mathrm{c}} = \gamma/E\,d$.

Concluding the discussion let me remark that the instability of the skin rippling is related to the critical load of Euler [4]. While in Euler's formula a stick becomes unstable with a wavelength of $\lambda = 2\,L$ the skin, because of the gravity term, becomes unstable with the essentially smaller wavelength λ_{c}.

Finally we report first experimental results which prove that the relation in Eq. (5) is fulfilled. Plastic foils of Hostaphan (Kalle) [5] layed on a surface of destilled water acquire a periodic ripple by compressive strain. Figure 8 is a double logarithmic

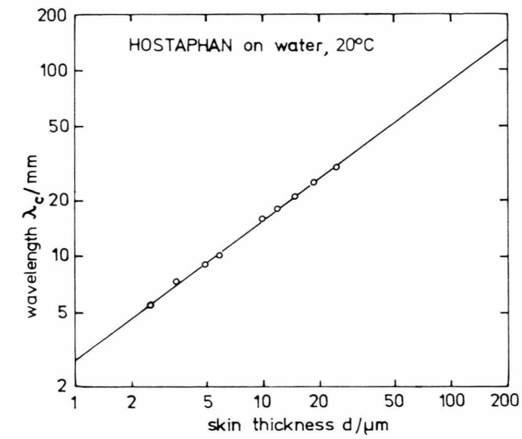

Fig. 8. Critical wavelength λ_{c} versus foil thickness d in a double logarithmic plot. Experimental points give $\lambda_{\mathrm{c}} = 87 \cdot d^{3/4}\,m^{1/4}$ invery good agreement with Equation (5).

plot of λ_{c} vs. d. The straight line has the form $\lambda_{\mathrm{c}} = C \cdot d^{3/4}$. It excellently fits to the experimental points by using $C = 87\ \mathrm{m}^{1/4}$. With $E = 4.5 \times 10^{9}$ N/m^2 from producers data sheet one calculates $C = 87.9\ \mathrm{m}^{1/4}$ which again is in very good agreement with the experiment.

[1] R. F. Rolsten, J. Appl. Phys. **46,** 4784 [1975].
[2] see for instance A. E. Scheidegger, Hdb. d. Physik, edit. S. Flügge, Bd. XLVII, 267 ff. [1956].
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[4] see for instance J. W. Geckeler, Hdb. d. Physik, edit. H. Geiger and K. Scheel, Bd. VI, 277 ff. [1928].
[5] Trade name for plastic foils made out of polyethylene terephthalate. The Kalle (Hoechst) AG, Wiesbaden, kindly supplied us with foils of type RN and RE.